BOOK

A Simplified Approach
to
Data Structures

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EXPRESSION TREE
&
HUFFMAN ALGORITHM

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CONTENTS FOR TODAY’S LECTURE

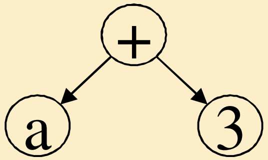
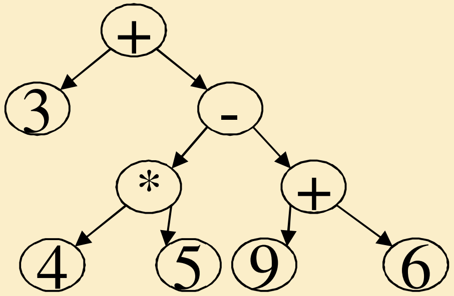
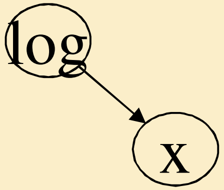
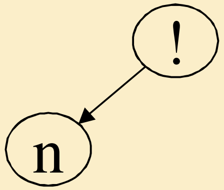
- Introduction to Expression trees
- Why Expression Tree
- Implementation of Expression trees
- Introduction to Huffman Algorithm
- Prefix Codes
- Huffman Code : Construction
- Huffman Code : Decoding

EXPRESSION TREES

- An expression tree for an arithmetic, relational, or logical expression is a binary tree in which :

- The parentheses in the expression do not appear.
- The leaves are the variables or constants in the expression.
- The non-leaf nodes are the operators in the expression :
  - A node for a binary operator has two non-empty subtrees.
  - A node for a unary operator has one non-empty subtree.

Example of Expression Tree

Expression	Expression Tree	Inorder Traversal Result
(a+3)		a + 3
3+(4*5-(9+6))		3+4*5-9+6
log(x)		log x
n!		n !

Why Expression Trees?

- Expression trees are used to remove ambiguity in expressions.
- Consider the algebraic expression 2 - 3 * 4 + 5.
- Without the use of precedence rules or parentheses, different orders of evaluation are possible :

((2-3)*(4+5)) = -9
((2-(3*4))+5) = -5
(2-((3*4)+5)) = -15
(((2-3)*4)+5) = 1
(2-(3*(4+5))) = -25

- The expression is ambiguous because it uses infix notation : each operator is placed between its operands.

Why Expression trees? (contd.)

- Storing a fully parenthesized expression, such as ((x+2)-(y*(4-z))), is wasteful, since the parentheses in the expression need to be stored to properly evaluate the expression.
- A compiler will read an expression in a language like Java, and transform it into an expression tree.
- Expression trees impose a hierarchy on the operations in the expression. Terms deeper in the tree get evaluated first. This allows the establishment of the correct precedence of operations without using parentheses.
- Expression trees can be very useful for:
  - Evaluation of the expression.
  - Generating correct compiler code to actually compute the expression's value at execution time.
  - Performing symbolic mathematical operations (such as differentiation) on the expression.

Implementing the Expression Tree

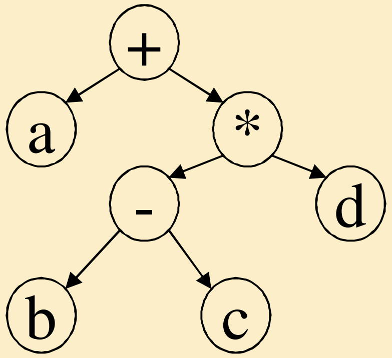
Expression Trees can be achieved by using three notations. These are :

- Prefix Notation
- Infix Notation
- Postfix Notation

Prefix Notation

- A preorder traversal of an expression tree yields the prefix (or polish) form of the expression.
- In this form, every operator appears before its operand(s).

For Example , Consider the tree :

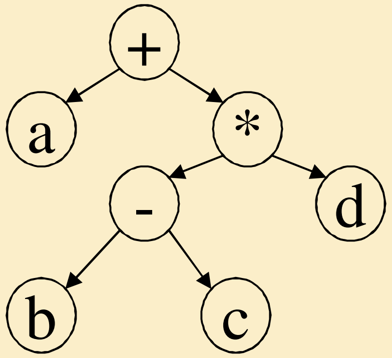


Prefix Notation : + a * - b c d

Infix Notation

- An inorder traversal of an expression tree yields the infix form of the expression.
- In this form, every operator appears between its operand(s).

For Example , Consider the tree :

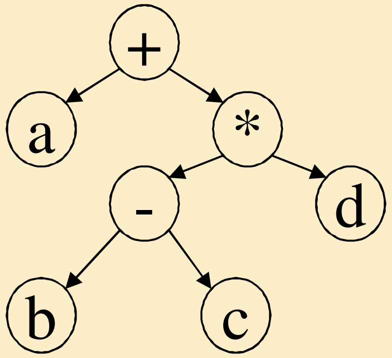


Infix Notation : a + b - c * d

Postfix Notation

- An postorder traversal of an expression tree yields the postfix form of the expression.
- In this form, every operator appears after its operand(s).

For Example , Consider the tree :



Postfix Notation : a b c - d * +

Prefix, Infix, and Postfix Forms (contd.)

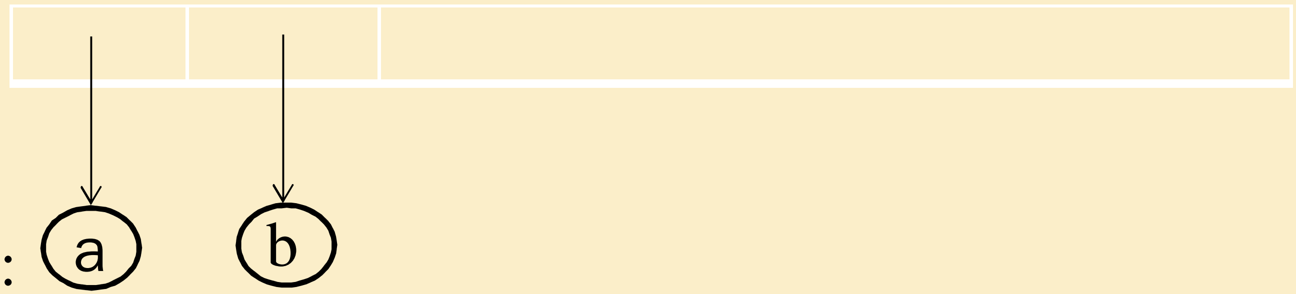
Expression	Prefix forms	Infix forms	Postfix forms
(a + b)	+ a b	a + b	a b +
a - (b * c)	- a * b c	a - b * c	a b c * -
log (x)	log x	log x	x log
n !	! n	n !	n !

Expression Tree from Postfix Notation

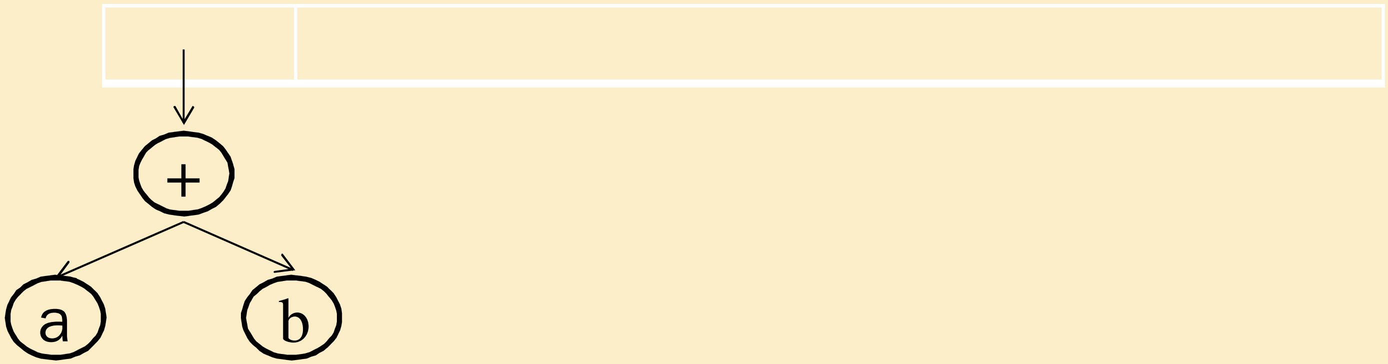
Consider the expression (a + b) * c. The postfix expression is:

a b + c *

Step 1 :

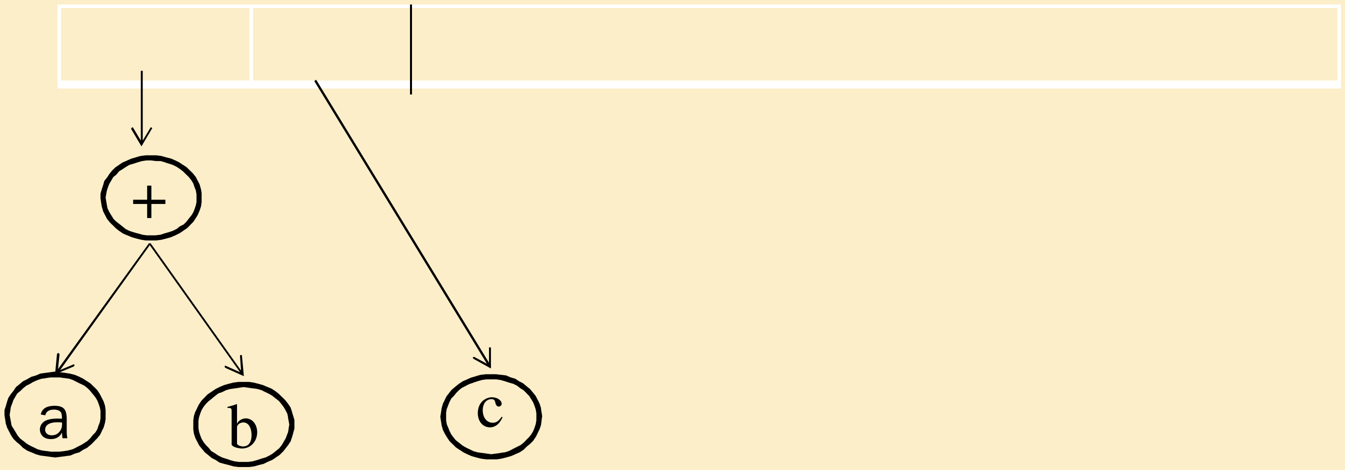


Step 2 :

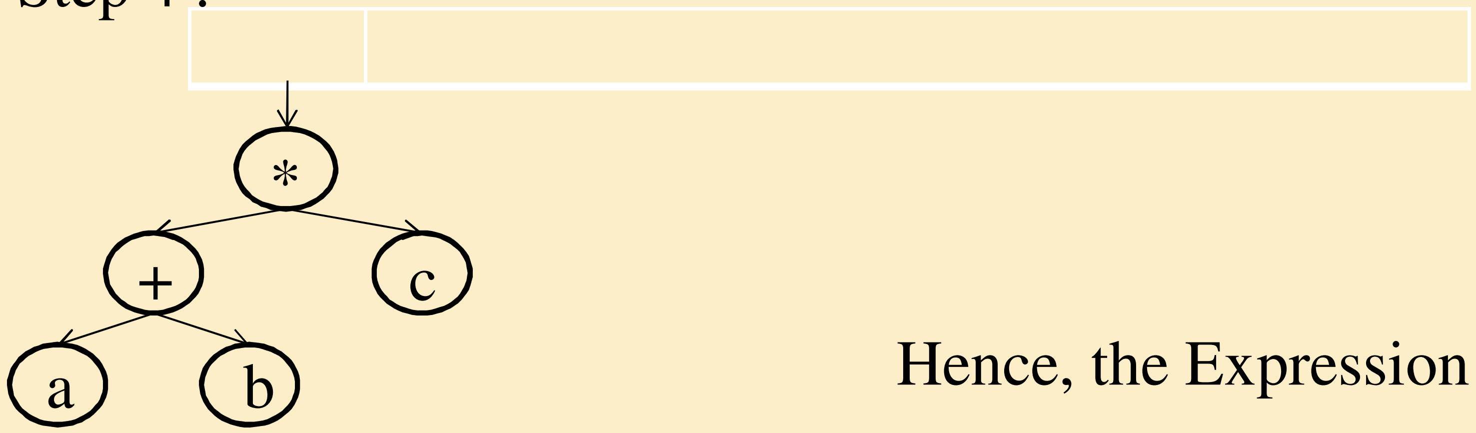


Expression Tree from Postfix Notation

Step 3 :



Step 4 :



Hence, the Expression Tree.

NOTE : For a computer generator program constructing an expression tree from infix notation is not preferred. Instead , a computer program uses postfix expression to express the expression tree. Because in postfix expression there is no need to apply rules of precedence and associativity.

HUFFMAN ALGORITHM

MOTIVATION

- Suppose we want to store and transmit very large files (messages) consisting of strings (words) constructed over an alphabet of characters (letters).
- Representing each character with a fixed-length code will not result in the shortest possible file!
- Example: 8-bit ASCII code for characters

– some characters are much more frequent than others

– using shorter codes for frequent characters and longer ones for infrequent ones will result in a shorter file.

Coding : Problem Definition

- Represent the characters from an input alphabet using a variable-length code alphabet C, taking into account the occurrence frequency of the characters.
- Desired properties:
  - The code must be uniquely decipherable: every message can be decoded in only one way.
  - The code must be optimal with respect to the input probability distribution.
  - No string is a prefix of another.

Example

Character	a	b	c	d	e	f
Frequency (%)	45	13	12	16	9	5
Fixed Length	000	001	010	011	100	101
Variable Length	0 101	101	100	111	1101	1100

Message: abadef

Fixed Length : 000001000011100101

Variable Length : 0101011111011100

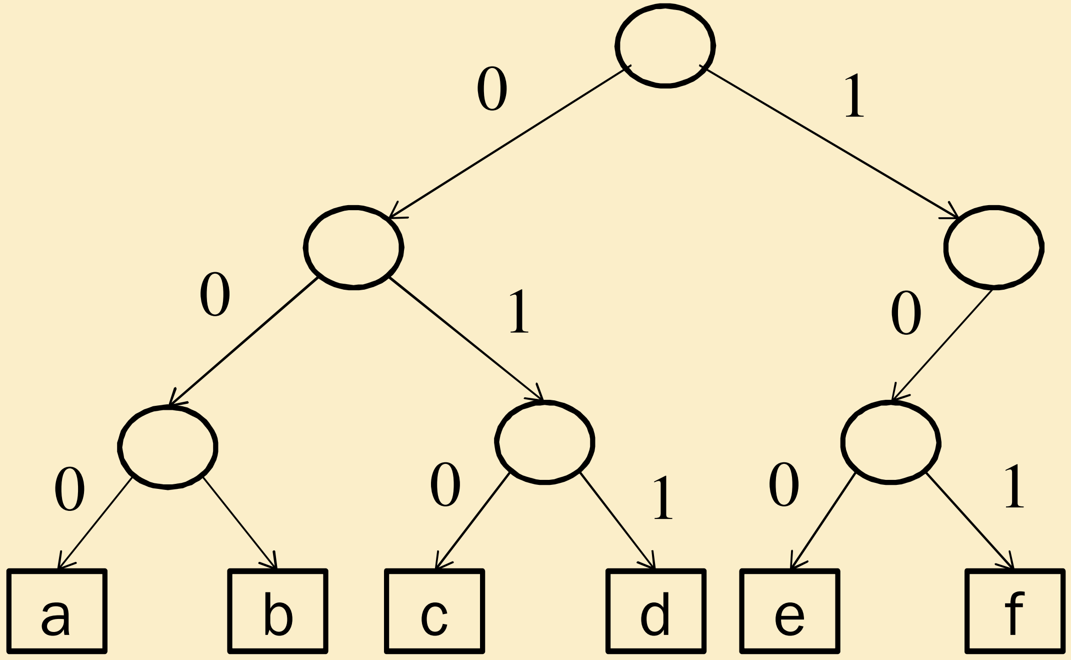
A file of 100,000 characters takes:

- 3×100,000 = 300,000 bits with fixed-length code
- (.45×1 + .13×3 + .12×3+ .16×3 + .09×4 + .05×4) ×100,000 = 224,000 bits on average with variable-length code (25% less)

Prefix Codes

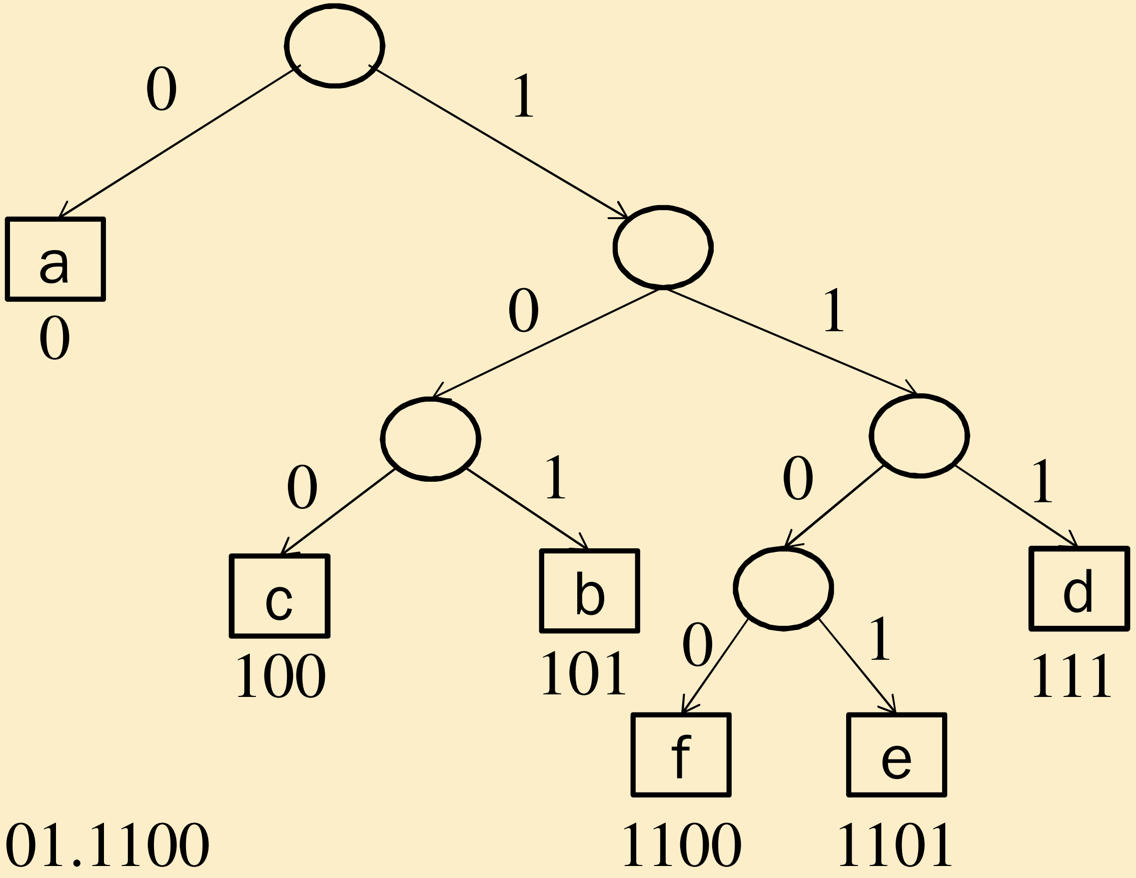
- We consider only prefix codes: no code-word is a prefix of another code-word. Prefix codes are uniquely decipherable by definition.
- A binary prefix code can be represented as a binary tree:
  - leaves are a code-words and their frequency (%)
  - internal nodes are binary decision points: “0” means go to the left, “1” means go to the right of a character. They include the sum of frequencies of their children.
  - The path from the root to the code-word is the binary representation of the code-word.

Example: fixed-length prefix code (1)



Message: 000.001.000.011.100.101 abadef

Example: variable-length prefix code (2)



Message: 0.101.0.111.1101.1100

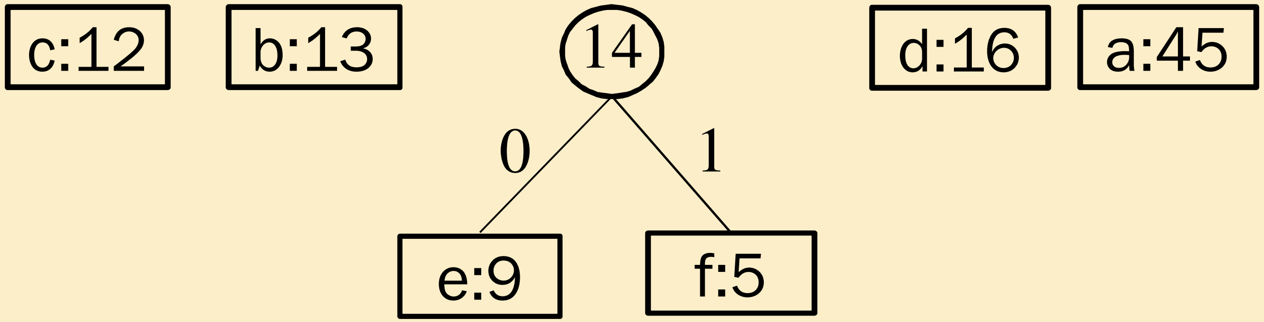
Huffman code: construction

- Idea: build the tree bottom-up, starting with the code-words as leafs of the tree and creating intermediate nodes by merging the new object whose frequency is the sum of the frequencies of the merged objects.
- To efficiently find the two least-frequent objects, use a minimum priority queue.
- The result of the merger of two objects is a new object whose frequency is the sum of the frequencies of the merged objects.

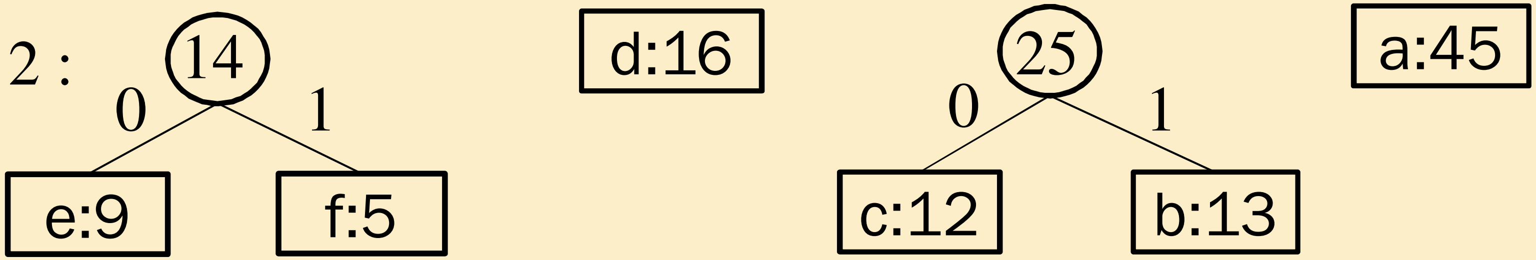
Example : Huffman Code Construction(1)

Start : f:5 e:9 c:12 b:13 d:16 a:45

Step 1 :

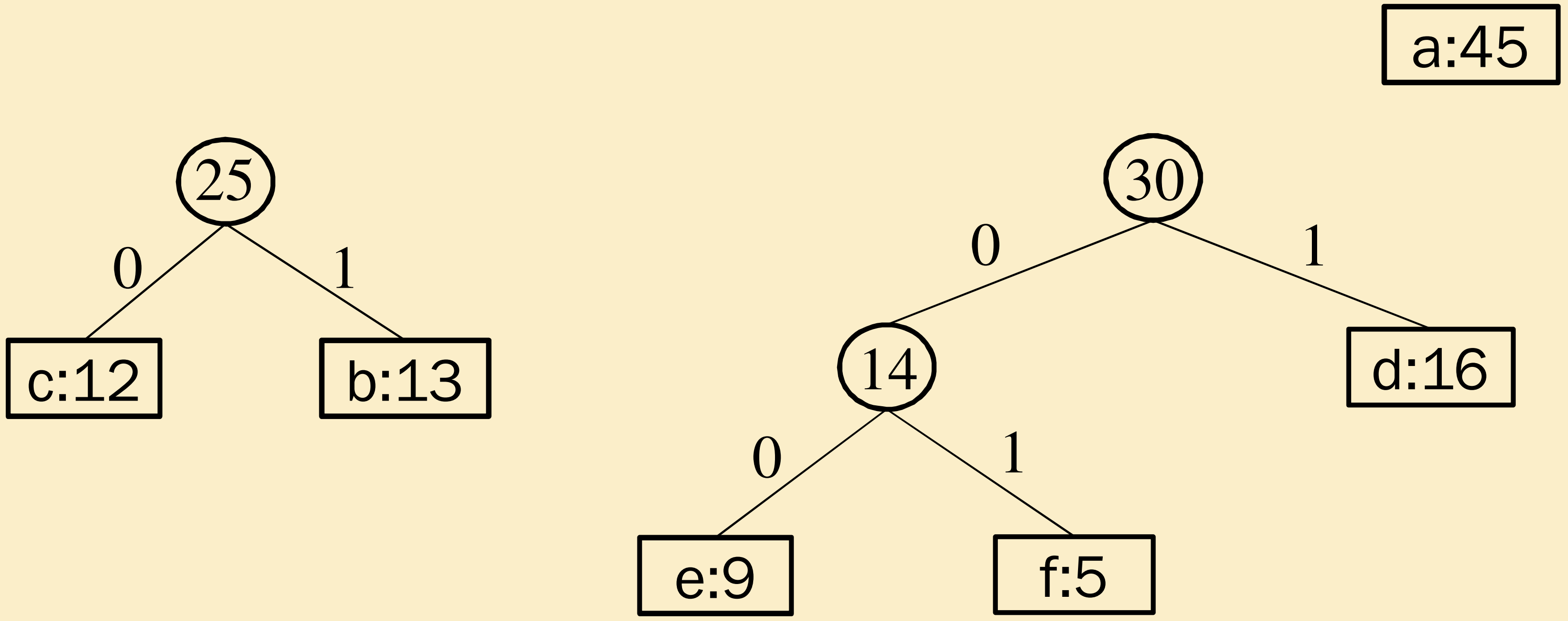


Step 2 :



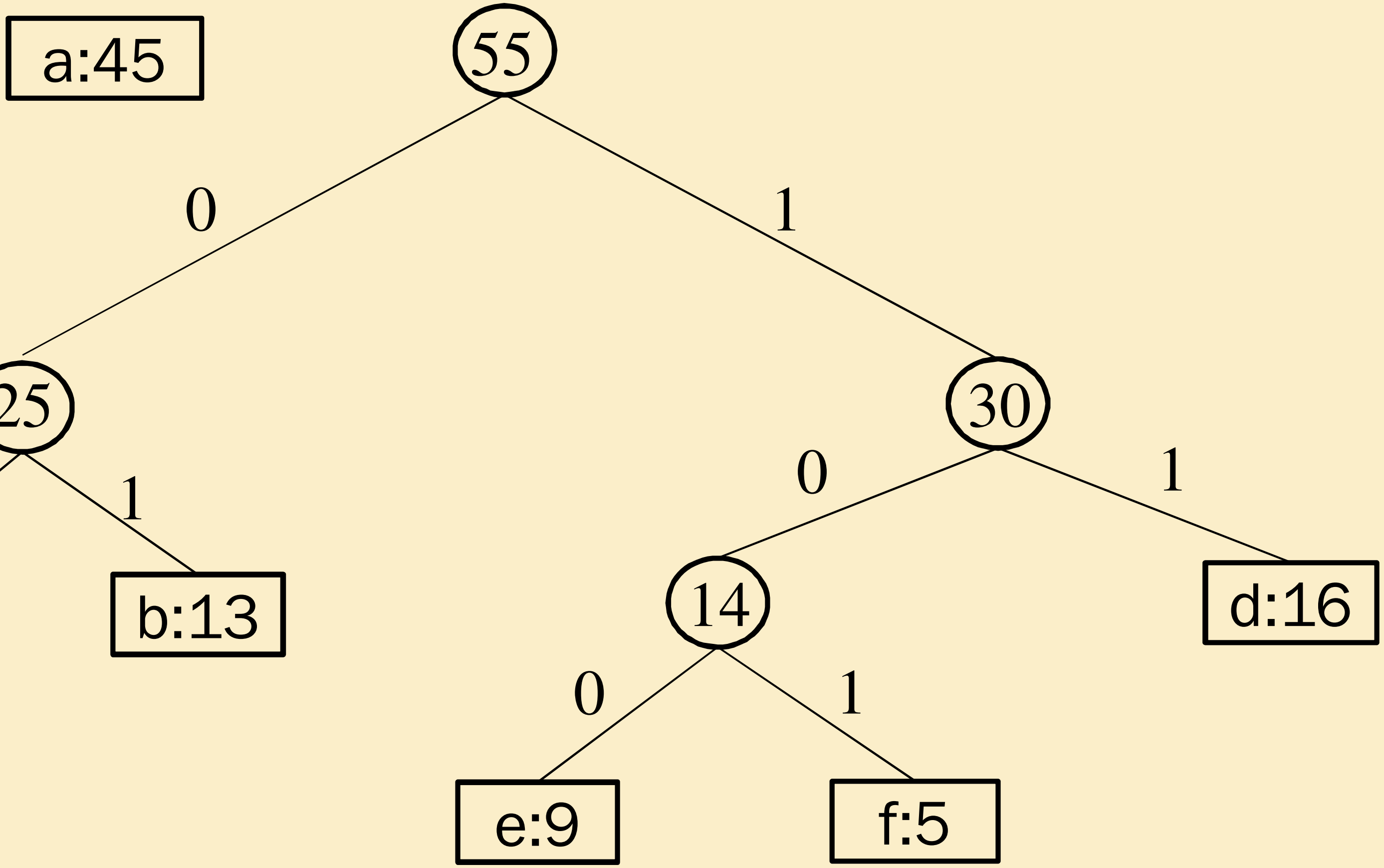
Example : Huffman Code Construction(2)

Step 3:



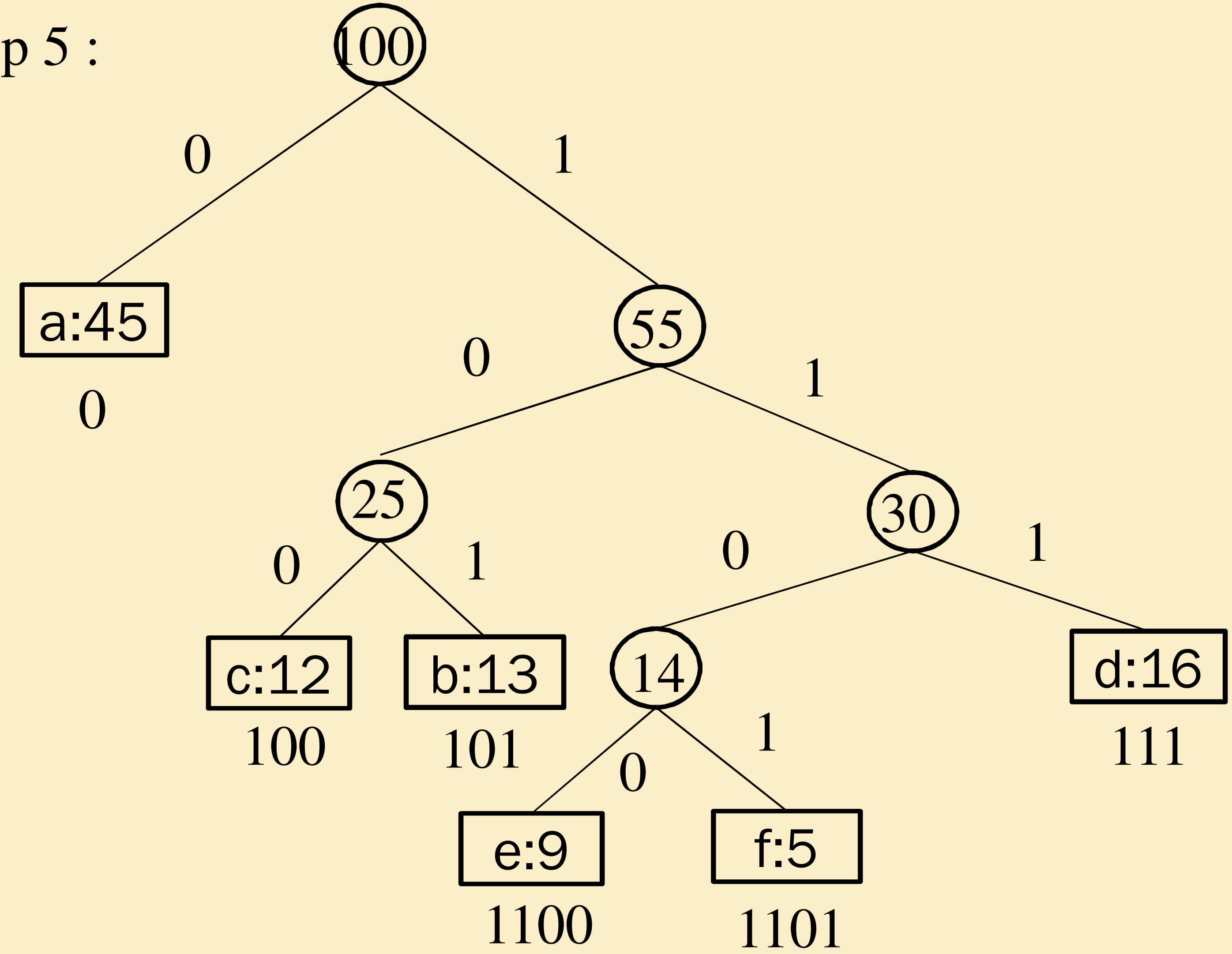
Example : Huffman Code Construction(3)

Step 4 :



Example : Huffman Code Construction(4)

Step 5 :



Example : Huffman Code Construction(5)

Result : Codes for the variables :-

a : 0

b : 100

c : 101

d : 111

e : 1100

f : 1101

Hence, no code is the prefix of another code.

Huffman code: decoding

- Huffman invented in 1952 a greedy algorithm for constructing an optimal prefix code, called a Huffman code.
- Decoding:
    1. Start at the root of the coding tree T, read input bits.
    2. After reading "0" go left
    3. After reading "1" go right
    4. If a leaf node has been reached, output the character stored in the leaf, and return to the root of the tree.

Complexity: O(n), where n is the message length